Renyi Entropy of MPO

\[ \rho = \begin{array}{cccccc}
\hline
& & & & & \\
\hline
\end{array} \]

Let a state (density matrix) be represented as an MPO

Say region \( A \) is first 3 sites

\[ \begin{array}{cccccc}
\hline
& & & & & \\
\hline
\end{array} \]

Trace over \( B \) to get reduced density matrix \( \rho_A \)

\[ \rho_A = \begin{array}{cccccc}
\hline
& & & & & \\
\hline
\end{array} \]

\[ = \begin{array}{cccccc}
\hline
& & & & & \\
\hline
\end{array} \]

absorb into last tensor
So $\rho_A$ is also an MPO

Renyi entropy is

$$S_2 = -\ln \text{Tr} \left[ \rho_A^2 \right]$$

$$= -\ln \begin{bmatrix}
  \cdots \\
\end{bmatrix}$$

**Technical notes:**

- Scaling of this method is $K^3$ where $K$ is MPO bond dimension.

- When tracing and squaring the MPO tensors, it may be best to factor out norms of each tensor and save log of each norm, then including in final result otherwise $\text{Tr}[\rho_A^2]$ could be very large (possible overflow)